STAT 2593

Lecture 030 - The Z Test for Hypotheses about a Population Mean

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The Z Test for Hypotheses about a Population Mean

Learning Objectives

1. Understand how we test population means in (approximately) normally distributed populations.

See my past discussion of introductory statistics education.

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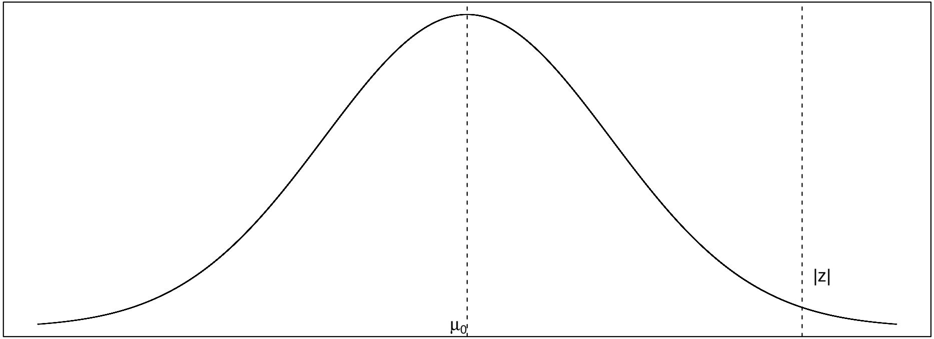
- How do we find our p-value?

Calculating p-values



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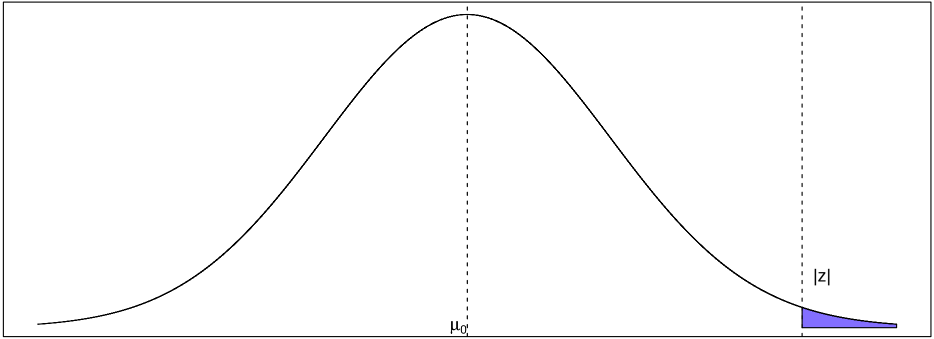

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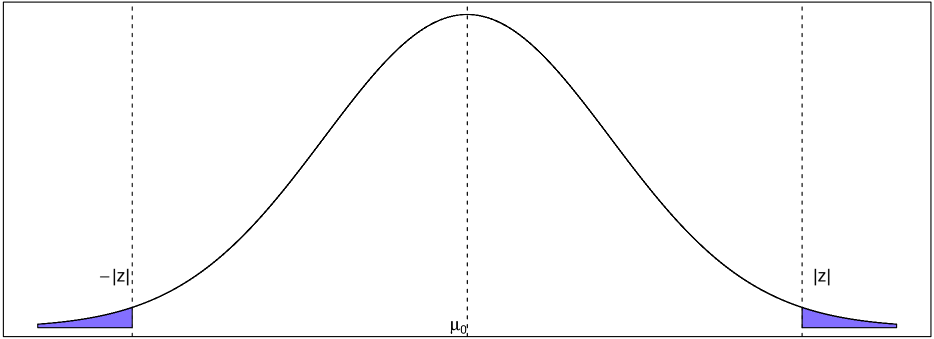

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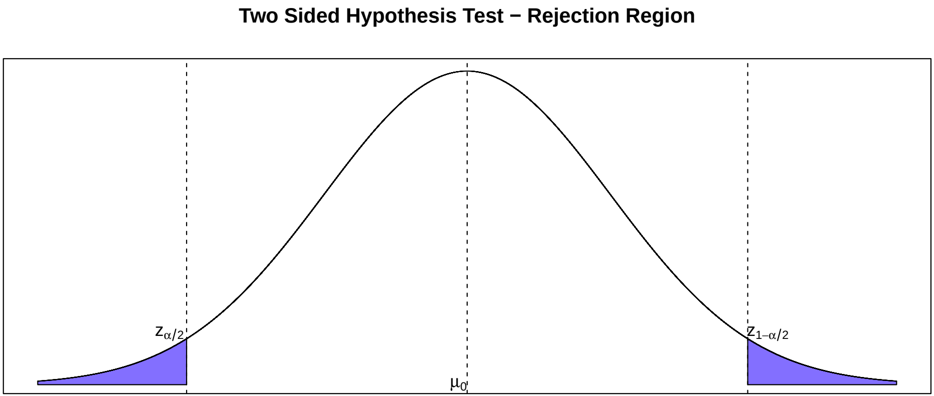
  - Note here we do not take the absolute value.

Rejection Regions for Hypothesis Tests - Critical Values



Summary

- If the sampling distribution is approximately normally distributed, can use a $N(0, 1)$ to run hypothesis tests.

- The rejection region depends on the alternative being considered.